University of California, Berkeley
Physics H7A Fall 1998 (*Strovink*)

SOLUTION TO PRACTICE EXAMINATION 1

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1. For each of parts (a.) through (e.), specify magnitude, unit, and direction. Giving vector components is sufficient for magnitude and direction. The mass is 4 kg, the initial velocity is 3 meters per second, and the initial acceleration is 8 meters per second^2. The force acting on the particle is constant, so the acceleration is constant.

(a.) The initial momentum is $\mathbf{p} = m\mathbf{v} = 12$ kg m/sec, in the $+\hat{\mathbf{y}}$ direction.

(b.) The force acting on the particle can be found by $F = ma$, which is 32 kg m/sec^2, or newtons, in the $+\hat{\mathbf{x}}$ direction.

(c.) At $t = 0.375 = 3/8$ sec, the momentum can be found easily because the acceleration is constant. The velocity in the $+\hat{\mathbf{y}}$ direction isn't changing because there is no force, but the $+\hat{\mathbf{x}}$ velocity is given by $v_x = at = 3$ m/sec. The momentum vector is thus (12,12) kg-m/sec. Alternatively, we can write this as a magnitude and a unit vector. The magnitude of this vector is $12\sqrt{2}$. The unit vector must have equal x and y components, and its length must be one. This gives $\hat{\mathbf{p}} = (\hat{\mathbf{x}} + \hat{\mathbf{y}})/\sqrt{2}$.

(d.) The position at $t = 1$ sec is given by the formulas for constant acceleration. In the y direction, there is no acceleration, so y is given simply by $y = y_0 + v_{0y}t = 6$ m. In the x direction there is a constant acceleration, so the position is given by $x = x_0 + v_{0x}t + at^2/2 = 8$ m. The position vector is thus (8,6) m. Alternatively, the length of this vector is 10 meters; its direction is given by the unit vector $\hat{\mathbf{r}} = (0.8\hat{\mathbf{x}} + 0.6\hat{\mathbf{y}})$.

(e.) The y component of the velocity is constant because there is no force in the y direction. The x component of the velocity vanishes at $t = 0$. Therefore $|\mathbf{v}| = \sqrt{v_x^2 + v_y^2}$ can never be smaller than it is at $t = 0$, when it is 3 m/sec in the $\hat{\mathbf{y}}$ direction.

2. An oil drop has mass m and charge q. It moves between plates a distance d apart, and voltage V is applied. The electrical force on the drop is qV/d upwards. When the drop is moving, it encounters a drag force $\mathbf{F} = -k\mathbf{v}$.

(a.) At $t < 0$ the drop is stationary. The electrical force must balance the gravitational force, so $mg = qV/d$, giving the potential V

$$V = \frac{mgd}{q}$$

(b.) The plates are shorted at $t = 0$, so there is no more electrical force. At this instant, the drop isn't moving, so it feels only the gravitational force. Its acceleration is just the acceleration of gravity, $a = g$ downwards.

(c.) As $t \to \infty$, the acceleration goes to zero. The velocity can be found by balancing the drag force with the gravitational force, $kv = mg$, so the terminal velocity is

$$v(t \to \infty) = \frac{mg}{k}$$

(d.) For $t > 0$ we can find a differential equation for the velocity. Newton's second law states that $F = dp/dt$ which in this case can be written $F = m\,dv/dt$. There are two forces, gravity and the drag force. The equation we get is

$$m\frac{dv}{dt} = mg - kv$$

(e.) Following the hint, we take d/dt of the answer for (d.):

$$m\frac{d^2v}{dt^2} = -k\frac{dv}{dt}$$

Substituting $a = dv/dt$:

$$m\frac{da}{dt} = -ka$$

Rearranging:
$$\frac{da}{a} = -\frac{k}{m}dt$$
Integrating from 0 to t:
$$\ln a(t) - \ln a(0) = -\frac{k}{m}t$$
Exponentiating:
$$\frac{a(t)}{a(0)} = e^{-\frac{k}{m}t}$$
From part (b.), $a(0) = g$, so
$$a(t) = g\, e^{-\frac{k}{m}t}$$
The acceleration begins with value g and decreases exponentially with time constant equal to m/k.

3. Instantaneously after the collision of the bullet and block, after the bullet has come to rest but before the frictional force on the block has had time to slow it down more than an infintesimal amount, we can apply momentum conservation to the bullet-block collision. At that time the total momentum of the block+bullet system is $(M+m)v_0'$, where v_0' is the velocity of the block+bullet system immediately after the collision. Momentum conservation requires that momentum to be equal to the initial momentum mv of the bullet. Thus
$$v_0' = \frac{mv}{M+m} \ .$$
After the collision, the normal force on the block+bullet system from the table is $(M+m)g$, giving rise to a frictional force
$$\mu N = \mu(M+m)g$$
on the sliding block+bullet system. This causes a constant acceleration μg of that system opposite to its velocity.

Take $t = 0$ at the time of collision. Afterward, the block+bullet system's velocity in the horizontal direction will be $v'(t) = v_0' - \mu g t$. It will continue sliding until $v'(t) = 0$, at which point the frictional force will disappear and it will remain at rest. Solving, the time at which the block-bullet system stops is
$$t = v_0'/(\mu g) \ .$$
The distance traveled in that time is
$$x = v_0' t - \frac{1}{2}\mu g t^2 = \frac{1}{2}v_0' t = \frac{(v_0')^2}{2\mu g} \ .$$
Plugging in the already deduced value for v_0', this distance is
$$x = \left(\frac{m}{M+m}\right)^2 \frac{v^2}{2\mu g} \ .$$